

# QCD and Quark/Hadron Matter<sup>1</sup>

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## Abstract

A brief review on the recent theoretical progress in hot/dense QCD is given. Special emphasis is put on the non-perturbative aspects of QCD plasma and the modification of hadron properties near the critical temperature of chiral transition.

## 1 Introduction

Due to the asymptotic freedom, the QCD coupling constant  $g(\mu)$  decreases logarithmically as one increases the renormalization scale  $\mu$ ,

$$\frac{g^2(\mu)}{4\pi} \simeq \frac{12\pi}{(33 - 2N_f) \ln(\mu^2/\Lambda^2)} \quad . \quad (1.1)$$

Now,  $\mu$  should be chosen to be a typical scale of the system to suppress the higher orders in  $g$ . In extremely hot and/or dense QCD medium,  $\mu$  will thus be proportional to  $T$  (temperature) or the chemical potential, which indicates that the system is composed of weakly interacting quarks and gluons at high  $T$  and/or high baryon density  $\rho$  (quark-gluon plasma phase). On the contrary, quarks and gluons are confined inside mesons and baryons at low  $T - \rho$  (hadronic phase). Therefore one expects a phase transition between the two phases at certain  $T$  and  $\rho$ . This expectation is actually “proved” by the numerical simulations of QCD formulated on the lattice at finite  $T$  [1]. Experimental efforts to create and detect the quark-gluon plasma in the relativistic heavy-ion collisions have also been started at BNL and CERN and bigger projects are planned in these laboratories. In this report, I am going to discuss three topics, (i) an intuitive idea of the QCD phase transition, (ii) recent theoretical progress in hot/dense QCD and (iii) some interesting observables in future experiments.

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## 2 How does the QCD phase transition occur?

Imagine heating up the QCD vacuum. At low  $T$ , pions (the lightest mode in QCD) are thermally excited. Since the pion has its own size (radius  $\sim 0.6\text{fm}$ ), the thermal pions start to overlap with each other at certain temperature  $T_c$  and dissolve into a gas of quarks and gluons above  $T_c$ . One can estimate  $T_c$  by identifying (pion's volume) $^{-1}$  with the pion number density at  $T_c$  obeying the Bose-Einstein distribution  $n_\pi(T_c)$ ;

$$n_\pi(T_c) = \left[\frac{4}{3}\pi R_\pi^3\right]^{-1}, \quad (2.1)$$

which gives  $T_c \sim 200$  MeV. One can make similar estimate for cold but dense matter. The “critical” density  $\rho_c$  is evaluated as

$$\rho_c = \left[\frac{4}{3}\pi R_N^3\right]^{-1} \simeq 3\rho_0, \quad (2.2)$$

with  $\rho_0$  being the normal nuclear matter density ( $0.17\text{ fm}^{-3}$ ).

$T_c$  and  $\rho_c$  are not too far from the typical hadronic scale, which gives us a hope to create quark-gluon plasma in the laboratory experiments (such as RHIC at BNL and LHC at CERN). Other than the relativistic heavy-ion collisions, the phase transtion is believed to take place in the early universe (roughly  $10^{-5}$  sec after the big bang) and also it may exist in the deep interior of the neutron stars. One should note here that we have so far used the word “phase transtion” in a loose sense. Whether the transition to the quark-gluon plasma is associated with a rapid change of a well-defined order parameter or not is still a controversial matter as we will see later.

## 3 Recent topics in hot/dense QCD

### 3.1. Multi scale structure at $T \gg T_c$

It is now widely believed that there exists non-perturbative physics in the infrared scale even if  $T$  is much larger than  $T_c$  [2]. This aspect is summarized as the following three scale structure:

$$g^2T < gT < T, \quad (3.1)$$

with  $g$  being the running coupling constant at finite  $T$ .  $gT$  is a scale of the electric screening mass of gluons and  $g^2T$  is related to the magnetic screening mass. As far as one stays in low orders of  $g$ , we have a consistent perturbation theory for static

and dynamical properties of the system. On the other hand, once one tries to go to higher orders or to look at the system by a probe of energy scale of  $O(g^2T)$ , the perturbation theory starts to break down, e.g. at  $O(g^6)$  for the free energy and at  $O(g^4)$  for the magnetic mass [2]. In this case, one has to sum up infinite numbers of diagrams to get sensible results, which is certainly a non-perturbative task.  $g^2T$  is also related to the coupling constant of 3D QCD as a high  $T$  effective theory of 4D QCD; the non-perturbative nature of 4D QCD at  $O(g^2T)$  has close relation to the confining nature of 3D QCD [3].

As an intuitive tool to understand the coexistence of non-perturbative (**NP**) and perturbative (**P**) physics at high  $T$ , let us introduce a scale  $K$  in momentum space which separates the **NP** and **P**. (See Fig.1.) Since the **NP** region is limited in a finite volume in momentum space, it will not affect the bulk properties of the system for high enough  $T$  where the typical frequency of quarks and gluons is  $(\text{a few}) \times T \gg K$ . However, if one probes low frequency region or decreases  $T$  toward  $T_c$ , effect of the **NP** region emerges. This feature is actually seen in the lattice QCD simulations of the energy density  $\mathcal{E}$  and pressure  $\mathcal{P}$  [4], and also the hadronic screening masses [5]. Teiji Kunihiro and myself have predicted the non-perturbative phenomena in the hadron spectra above  $T_c$  in the scalar and pseudoscalar channels [6] before the appearance of the lattice data.

Fig. 1: Separation of scales at high  $T$ .

### 3.2. Order of the phase transition

The determination of the order of the QCD phase transition near  $T_c$  is one of the central issues of the recent lattice QCD study. As for the pure gauge system without dynamical fermion ( $m_q = \infty$ ), the center symmetry ( $Z(3)$  in  $SU_c(3)$  case) controls the confinement-deconfinement phase transition. The effective  $Z(3)$  spin model predicts the 1st order transition and the lattice studies with finite scaling analyses support this feature [7]. Although it is of 1st order, the transition is much weaker than that seen before on the smaller lattice. Once one introduces dynamical fermions,  $Z(3)$

symmetry is explicitly broken. However, as far as  $m_q$  is large enough, one can still study the phase transition based on this approximate symmetry. On the other hand, in the opposite limit where  $m_q$  is zero, chiral symmetry instead of  $Z(3)$  symmetry takes place to control the phase transition with  $\langle \bar{q}q \rangle_T$  as an order parameter. For finite quark masses ( $m_{u,d} = O(10\text{MeV})$  and  $m_s = O(200\text{MeV})$ ), chiral symmetry is explicitly broken again, but one can still study the phase transition based on the approximate chiral symmetry. The recent lattice data [8] show, although the chiral transition near  $m_q = 0$  seems to be 1st order, the phase transition is not observed for the realistic values of  $m_{u,d,s}$ . However, this conclusion could be changed in the future large scale simulations with finite size scaling analysis.

The order of the chiral transition is most relevant to the big-bang nucleosynthesis of  $^9\text{Be}$ ,  $^{10}\text{B}$  and  $^{11}\text{B}$  [9]. The spacial inhomogeneity due to the bubble formation during the 1st order chiral transition can create those relatively heavy elements, while the standard homogeneous model creates only 10-100 smaller abundances.

From the point of view of the relativistic heavy-ion collisions, the precise order of the transition is not much relevant because the system size is finite. Instead the global behavior of the order parameter and the entropy as a function of temperature are rather important for the time-evolution of the system and for related experimental signals of the formation of the quark-gluon plasma.

The energy density and entropy at finite  $T$  are known to have rapid growth in a narrow range of temperature ( $\sim 10$  MeV) by the numerical simulations on the lattice. As for the quark condensates at finite  $T$ , both lattice QCD and QCD effective lagrangians predict (i) a rapid change of the light quark condensate around  $T_c$  and (ii) a smooth change of the strange condensate across this temperature [10]. (See Fig. 2.) This might indicate a considerable difference of the behavior of the hadronic matter at finite  $T$  in the  $u, d$  sector and that in the  $s$  sector.

Fig. 2: Light quark condensate (solid line) and the strange-quark condensate (dashed line) at finite

$T$  in an effective theory of QCD [10].

### 3.3. Dynamical critical phenomena

Since the quark condensate is a scale dependent quantity and is not an observable, one has to look for other physical quantity to see the signal of the chiral phase transition. The hadron masses at finite  $T$  is one of the possible candidates for such quantity. In fact, the masses of the light hadrons are essentially determined by the quark condensates as QCD sum rules tell us. This suggests that the mass shift of hadrons in medium will be a good measure of the partial restoration of chiral symmetry in medium [6, 11]. There exist similar situations in condensed matter physics: for example, the existence of the soft phonon mode is an indication that the ground state undergoes a structural phase transition. In QCD, scalar mesons (fluctuation of the order parameter) and the vector mesons (such as  $\rho$ ,  $\omega$  and  $\phi$ ) are the candidates of the “soft mode”. Su Houng Lee, Yuji Koike and myself have developed a method to calculate the mass shift of these mesons (QCD sum rules at finite temperature) [12, 13]. Both in scalar ( $\sigma$ ) and vector channels, one can establish a relation between  $m(T)$  (meson-mass at finite  $T$ ) and the 4-quark condensate  $\langle(\bar{q}q)^2\rangle_T$ .

$$m(T) \leftrightarrow \langle(\bar{q}q)^2\rangle_T. \quad (3.2)$$

The r.h.s. of (3.2) can be evaluated by the pion gas approximation, or hopefully by adopting the future lattice data. We found that, in the scalar and  $\rho$  channels, there is a sizable softening of the masses near  $T_c$ . (See, Fig.3.) Asakawa and Ko later generalized this approach to the  $\phi$  meson at finite  $T$  by taking into account the thermal strange particles to evaluate the r.h.s. of (3.2) and found a significant softening even in this channel [14]. (See, Fig.4.)

Fig. 3: Light scalar and vector mesons at finite  $T$  in the QCD sum rules [12, 13].

Fig. 4:  $\phi$  meson at finite  $T$  in the QCD sum rules [14].

The similar calculation can be also done for the system at finite density (but  $T = 0$ ) as has been shown by Su Houn Lee and myself [15]. One of the main differences of this system from the  $(T \neq 0, \rho = 0)$  system is the behavior of the quark condensates. For instance, the light quark condensate in the chiral limit decreases slowly at low temperature

$$\langle \bar{q}q \rangle_T / \langle \bar{q}q \rangle_0 = 1 - [(N_f^2 - 1)/N_f] T^2 / 12f_\pi^2 + \dots, \quad (3.3)$$

while it decreases linearly at low density

$$\langle \bar{q}q \rangle_\rho / \langle \bar{q}q \rangle_0 = 1 - \rho \cdot \Sigma_{\pi N} / f_\pi^2 m_\pi^2 + \dots, \quad (3.4)$$

with  $\Sigma_{\pi N} = (45 \pm 10)\text{MeV}$  being the  $\pi$ -N sigma term. If one extrapolates the latter formula to normal nuclear matter density  $\rho_0 = 0.17\text{fm}^{-3}$ , the condensate decreases almost 20–30%. Accordingly, rapid change of hadron masses at relatively low densities has been predicted [15]. (See, Fig. 5.) The Walecka model of nuclear matter also predicts the similar decrease for the *omega*-meson as was shown recently [16].

Fig. 5: The masses of  $\rho$ ,  $\omega$  and  $\phi$  mesons in nuclear matter [15].

Now, what will be the observable consequences of the softening phenomenon? Since the vector mesons can decay into lepton pairs,  $\rho$ ,  $\omega$  and  $\phi$  are the best particles to be looked at. In the case of  $\rho$ , the shift of the peak position or the smearing the  $\rho$ -peak could be observed in the future relativistic heavy-ion collisions (See [13], [17] and Fig. 6.). Also, there is an experimental proposal to detect the peak-shift through the lepton pairs [18] where  $\rho$ -mesons are created inside the heavy nuclei using tagged photons at INS-ES. As for  $\phi$ , one might be able to see a double peak structure in the lepton pair spectrum in the heavy-ion collisions [19]. This is due to a combined effect of the peak-shift and the large entropy jump at the phase transition point. (See, Fig. 7). There is also an experimental proposal to detect the modification of the  $\phi$ -meson through lepton and kaon pairs [20] where  $\phi$ -mesons are created in heavy nuclei by p-A collisions at KEK-PS.

Fig. 6: The smearing of the  $\rho$  peak due to the mass shift in the dilepton invariant mass spectrum in the heavy-ion collisions [17].  $T_f$  denotes the freeze-out temperature and the solid line is a result without the mass shift.

Fig. 7: The dilepton invariant mass spectrum at the central rapidity in the heavy-ion collisions [19]. The solid curve is a result of the initial temperature = 250 MeV and the dashed curve is a result without phase transition. Right two peaks correspond to  $\phi$  (double  $\phi$  peak) and the peak near 0.8 GeV corresponds to  $\omega$ .



Up to this point, we have assumed that the restoration of chiral symmetry always takes place in the central rapidity region of the heavy-ion collisions at extreme high energies. However, there is another interesting possibility in which a disoriented chiral condensate is produced [21]. This disoriented chiral condensate will eventually decays into the vacuum orientation through the emission of pions. However, because of the event by event large fluctuation of the direction of the disorientation, one might observe a large isospin fluctuation for the low momentum pions in the hadron-hadron and nucleus-nucleus collisions. Realistic calculations using the linear sigma-model has been started to check whether such phenomenon really occurs or not [22]. Also, this might have some relevance to the cosmic-ray event Centauro.

## 4 Summary

### (1) *Non-perturbative physics above $T_c$ :*

The higher order analyses of the perturbation theory have shown that there exists non-perturbative physics even above  $T_c$ . The lattice data of the free energy and also the screening mass in the scalar-pseudoscalar channel seem to support the coexistence of perturbative and non-perturbative physics above  $T_c$ . There are several ideas about the physics behind them, which include the non-perturbative cutoff, the gluon condensate above  $T_c$ , color singlet soft modes above  $T_c$  and so on. Clear physical understanding of them are called for.

### (2) *Chiral restoration and its associated phenomena:*

$\langle \bar{q}q \rangle$  for light quarks is a good order parameter near the chiral limit. There is little doubt about the existence of chiral transition at high  $T$  in the chiral limit and probably it is also true at high  $\rho$ . Like similar cases in condensed matter and nuclear physics, we can expect phenomena associated with this phase transition; in particular the modification of the elementary modes of excitations (hadrons) in medium. QCD sum rules and effective theories predict a sizable softening of the scalar and vector mesons in medium. Furthermore, one might be able to see such softening by looking at the dileptons in the future laboratory experiments.

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## References

- [1] Lattice 92, Nucl. Phys. B (Proc. Suppl.) **30** (1993) March 1993.
- [2] A. D. Linde, Phys. Lett. **96B** (1980) 289.  
D. J. Gross, R. D. Pisarski and L. G. Yaffe, Rev. of Mod. Phys. **53** (1981) 43.
- [3] M. Ishii and T. Hatsuda, in preparation.
- [4] J. Engels, J. Fingberg, F. Karsch, D. Miller and M. Weber, Phys. Lett. **252B** (1990) 625.
- [5] C. DeTar and J. Kogut, Phys. Rev. **D36** (1987) 2828.
- [6] T. Hatsuda and T. Kunihiro, Phys. Rev. Lett. **55** (1985) 158.  
T. Hatsuda and T. Kunihiro, Phys. Rep. (1994) to be published.
- [7] M. Fukugita, M. Okawa and A. Ukawa, Phys. Rev. Lett. **63** (1989) 1768, Nucl. Phys. **B337** (1990) 181.
- [8] F. R. Brown et al., Phys. Rev. Lett. **65** (1990) 2491.
- [9] R. Boyd and T. Kajino, Astroph. J. **359** (1990) 267.
- [10] T. Hatsuda and T. Kunihiro, Phys. Lett. **198B** (1987) 126.
- [11] G. E. Brown and M. Rho, Phys. Rev. Lett. **66** (1991) 2720.
- [12] T. Hatsuda, Y. Koike and S. H. Lee, Nucl. Phys. **B394** (1993) 221.
- [13] T. Hatsuda, Y. Koike and S. H. Lee, Phys. Rev. **D47** (1993) 1225.
- [14] M. Asakawa and C. M. Ko, Nucl. Phys. **A** (1994) in press.
- [15] T. Hatsuda and S. H. Lee, Phys. Rev. **C46** (1992) R34.
- [16] H. C. Jean, J. Piekarewicz and A. G. Williams, Florida State Univ. report FSU-SCRI-93-132 (1993).
- [17] F. Karsch, K. Redlich and L. Turko, Z. Phys. **C60** (1993) 519.
- [18] H. Shimizu, private communication.
- [19] M. Asakawa and C. M. Ko, Phys. Lett. **B** (1994) in press.
- [20] H. Enyo and J. Chiba, private communication.
- [21] J. D. Bjorken, *Baked Alaska*, report SLAC-PUB-6109 (1993).
- [22] e.g., S. Gavin and B. Muller, report BNL-GM-1 (1993).